Analyse system of queue in an office. Population of customers is very large so that we can consider it is an infinite one). Customers for the service arrive randomly following a Poisson process. The office can process customers at an average rate of five patients an hour (one at a 12 minutes).The service process is also Poisson. Customers are served at an average of four per hour (one at a 15 minutes). The office operates 12 hours a day.

First, all measures assume the process has been operating long enough for the probabilities resulting from the physical characteristics of the problem to have made themselves felt; that is, the system is in equilibrium.

Second, the utilisation of the system is $\rho = \lambda/\mu = 4/5 = 0{,}8 < 1$, thus we can count relationships as follows:

1) Probability of the system being empty - expected idle time of the system:

$$p_0 = (1 - \rho) = 1 - 0{,}8 = 0{,}2$$

On average, the office will be idle 20 percent of the time and busy 80 percent of the time.

2) The expected number in the system - both in waiting line and being serviced is:

$$L = \frac{\lambda}{\mu - \lambda} = \frac{4}{5-4} = 4$$

There will be an average of four persons in line and being serviced.

3) The expected number in the waiting line is:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{4.4}{5(5-4)} = \frac{16}{5} = 3{,}2$$

There will be an average of 3,2 people in waiting line.

3) The expected number in the waiting line is:

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There will be an average of 3,2 people in waiting line.

4) The average waiting time (in the queue) of an arrival is:

$$W_q = \frac{L_q}{\lambda} = \frac{L}{\mu} = \frac{\frac{16}{5}}{4} = \frac{4}{5} = 0{,}8$$

The average waiting time of an customer is 0,8 of an hour (e.g. 48 minutes).

5) The average time an arrival spends in the system (both waiting and in service) is:

$$W = W_q + 1/\mu = 0{,}8 + 1/5 = 1$$

The average time spent in the system - both in the waiting line and obtaining service - is one hour.

If we assume a 12 hour workday, there will be an average of 48 customers arriving per day, and the expected total lost time of customers waiting will be:

$$T = \lambda .12. W_q = 4.12.0{,}8 = 38{,}4.$$

There will be a cost associated with these 38,4 hours. Assume the cost to the society is $10 for each hour lost by a customer waiting. The average cost per day from waiting is

38,4.10 = $384

Suppose that we could in some fashion increase the service rate from five to six per hour and thereby decrease the average time spent in service from 12 minutes to 10 minutes. What would be the effect of this change?

With $\mu = 6$, the expected number in the queue is

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{4.4}{6(6-4)} = \frac{16}{12} = 1{,}33$$

instead of the 3.2 found above. The average wait for the patient is now:

$$W_q = \frac{L_q}{\lambda} = \frac{1{,}33}{4} = \frac{1}{3} = 0{,}33$$ (of an hour).

Before the change, each customer spent an average of 12 minutes being served and 48 minutes waiting. After the change, each patient will spend an average of 10 minutes being served, and 20 minutes waiting. This can be verified by computing the total time in the system:

$$W = Wq + 1/\mu = 1/3 + 1/6 = 1/2 \text{ (of an hour)}.$$

Each day there are 48 customers (4.12), and each customer will save one-half hour in total. At a cost of $10 per hour, the daily cost savings is:

48.1/2.20 = $480.

It would be worth $480 per day or $175200 per year to the society to increase the service rate to six customers at office per hour.